ASSAM UNIVERSITY SILCHAR

SYLLABI for Two-Year M.Sc. (Mathematics) Program

(Under the guidelines of NEP-2020)

Effective from 2022-2023

DEPARTMENT OF MATHEMATICS

ALBERT EINSTEIN SCHOOL OF PHYSICAL SCIENCES

Vision and Mission of the Department

Department of Mathematics

VISION

- To strive for higher academic and research excellence.

MISSION

- To attract motivated and talented students by providing a learning environment where they can learn and develop the mathematical and computational skills needed to formulate and solve real-world problems.
- To produce quality Mathematicians and scientists through high quality research who can serve the society in the field of science, technology, and allied branches.
- To Propagate and disseminate the knowledge among mass through extension activities.

Department of Mathematics:

Brief about the Department:

The Department of Mathematics started functioning in the year 1994. The Department offers M.Sc., M.Phil., and Ph.D. programs in Mathematics. Academic activities of the Department apart from teaching and research include activities like organizing Seminars, Conferences, Workshops, Refresher Courses, Winter Schools programs, etc. for serving teachers and students. The Departmental Library is sponsored by the National Board for Higher Mathematics (DAE), Mumbai. The Department is also sponsored by UGC (SAP) and DST (FIST). The academic programs are managed by a competent core faculty and backed by updated syllabi and a computer laboratory.

List of Programs offered by the Department:

Program	**Title of the Program**
M.Sc.	M.Sc., Mathematics
Ph.D. (Full time, Part-time)	Mathematics

M.Sc. in Mathematics

Programme Objectives (POs):

The main objectives of M.Sc. Mathematics programme are:

- To develop mathematical aptitude and the ability to think abstractly.
- To develop the ability to read, follow and understand mathematical text on their own.
- To train the students to communicate mathematical ideas in a lucid and effective manner.
- To apply their theoretical, mathematical knowledge to solve various real-life problems.
- To encourage the use of relevant software to develop the computational abilities and programming skills.

Programme Specific Outcomes (PSOs):

On successful completion of the M.Sc. mathematics programme, students will

- Have a strong foundation in the core areas of Mathematics.
- Be able to apply mathematical skills and logical reasoning for problem solving.
- Have a good knowledge and exposure to advanced and contemporary fields of Mathematics.
- Become competent to succeed in national level competitive examinations for taking up research and teaching.
- Have an excellent domain knowledge in their chosen elective subjects leading to research.

M.Sc. (Mathematics) - Course Structure

I-Year, I-Semester

S.No.	Course Code	Course Name	Description	Hours	Credits
1	MAT500	Orientation	Compulsory	---	NIL
2	MAT501	Basic Abstract Algebra	Core	4	4
3	MAT502	Ordinary Differential Equations	Core	4	4
4	MAT503	Basic Linear Algebra	Core	4	4
5	MAT504	Mathematical Methods	SEC	3	3
6	MAT505	Basic Mathematical Software Lab	ALIF	6	3
7	MAT506	Compulsory Community Engagement Course	CCEC	2	2
			Total	**23**	**20**

I-Year, II-Semester

S.No.	Course Code	Course Name	Description	Hours	Credits
1	MAT551	Basic Analysis	Core	4	4
2	MAT552	Basic Operations Research	Core	4	4
3	MAT553	Partial Differential Equations	Core	4	4
4	MAT554	Introduction to Graph Theory	IDC	3	3
5	MAT555	Numerical Analysis (Theory & Lab)	ALIF	4	3
6	MAT556	Evolution of Modern Indian Mathematics	VBC	2	2
			Total	**23**	**20**

II-Year, III-Semester

S.No.	Course Code	Course Name	Description	Hours	Credits
1	MAT601	(A) Topology	Elective-I	4	4
		(B) Number Theory			
		(C) Functional Analysis			
		(D) Numerical Linear Algebra			
2	MAT602	Complex Analysis	IDC	4	4
3	MAT603	(A) Advanced Operations Research-I	Elective-II	4	4
		(B) Methods of Applied Mathematics			
		(C) Differential Geometry			
		(D) Calculus of Variations and Integral Equations			
4	MAT604	Mathematical Modelling (with Field Work)	ALIF	6	3
5	MAT605	Research Project-I	Dissertation	10	5
			Total	**28**	**20**

II-Year, IV-Semester

S.No.	Course Code	Course Name	Description	Hours	Credits
1	MAT651	Advanced Analysis	Core	4	4
2	MAT652	Mechanics & Applied Fluid Dynamics	Core	4	4
3	MAT653	(A) Commutative Algebra (B) Operator Theory (C) Cryptography (D) Advanced Computational Methods (E) Advanced Operations Research-II (F) Bio-Mathematics	Elective-III	4	4
4	MAT654	Research Project-II	Dissertation	16	8
			Total	**28**	**20**

DETAILED SYLLABUS FOR EACH COURSE

M.Sc. (Mathematics): I Year I Semester

Course Code: MAT501	Basic Abstract Algebra	L	P	C
		4	0	4

Pre-requisites: Definitions, examples and simple properties of groups and rings.

Course Objective: The main objective of this course is

1. To get the students introduced to abstract concepts in mathematics including groups, rings, and fields.
2. To encourage the students to appreciate various generalisations of the concepts of the number system and Euclidean geometry they are already familiar with.
3. To develop the logical thinking of the students.

Syllabus:

Unit-I:
Symmetric group, Lagrange theorem and applications, normal subgroup and group homomorphism, quotient group, external direct product of groups.

Unit-II:
Internal direct product, group action, orbit-stabiler theorem, Sylow's theorems with applications, class equation of finite groups.

Unit-III:
Prime ideal and maximal ideals, ring homomorphism, polynomial ring, principal ideal domain, Euclidean domain, finite fields.

Unit-IV:
Prime and irreducible elements, unique factorisation domain, Gauss' lemma, Eisenstein's irreducibility criterion and applications, finite and algebraic extension.

Unit-V:
Geometric constructions, Kroneker's theorem, splitting field, characterisations of normal extensions and separable extensions**,** fundamental theorem of Galois theory and its applications.

Recommended Text Books:

1. J. Gallian, *Contemporary Abstract Algebra*, Narosa Publishers.
2. V.K. Khanna, S.K. Bhambri, *A Course in Abstract Algebra*, 4^{th} edition.
3. D.S. Dummit, D.S., and R.M Foote, *Abstract Algebra*, John Wiley and Sons.

Course Outcomes: On successful completion of the course, students will be able to

1. Learn basic concepts in group theory, ring theory, field extensions and Galois theory.
2. Take up more advanced papers like commutative algebra, algebraic geometry and algebraic number theory.
3. Answer the questions related to Abstract Algebra in various competitive examinations including NET/GATE/NBHM examinations.

Course Code: MAT502	Ordinary Differential Equations	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objective: The aim of this course is to introduce ordinary differential equations and fundamental theorems for existence and uniqueness of their solutions. This course further explains the analytic techniques in computing the solutions of various ordinary differential equations, and the stability of linear and non-linear autonomous systems.

Syllabus:

Unit-I: First Order Differential Equations

Ordinary Differential Equations, Some mathematical models, First order equations, Existence and Uniqueness of solutions, Picard's and Peano's Theorems, Continuous dependence on initial conditions, Gronwall's inequality.

Unit-II: Second Order Linear Differential Equations

Wronskian, Explicit methods to find solutions, Method of variation of parameters, Sturm-Liouville equations, Eigenvalues and Eigenfunctions of Sturm-Liouville equations, Eigenfunction expansions, Green's functions.

Unit-III: Power Series Solutions and Special Functions

Power series solutions: Ordinary points, Regular singular points, Irregular singular points, Frobenius methods; Special functions: Legendre and Bessel functions, Properties.

Unit-IV: System of Differential Equations

Existence and uniqueness theorems, Homogeneous linear systems, Fundamental matrix, Exponential of a matrix, non-Homogeneous linear systems, linear systems with constant coefficients.

Unit-V: Non-linear Theory

Autonomous systems, Types of critical points, Stability of linear systems with constant coefficients, Stability of nonlinear systems.

Recommended Text Books:

1. Coddington, E.A., *An Introduction to Ordinary Differential Equations*, Dover Publications, 1989.

References:

1. Ross, S.L., *Introduction to Ordinary Differential Equations*, John Wiley & Sons, New York, 1989.
2. Nandakumaran, A.K, Datti, P.S. & Raju K.G., *Ordinary Differential Equations: Principles and Applications*, Cambridge University Press, 2018.
3. Deo, S.G. & Raghavendra, V., *Ordinary Differential Equations*, Tata McGraw Hill, New Delhi, 2006.
4. Simmons, G.F., *Differential Equations with Applications and Historical Notes*, Tata McGraw Hill, New Delhi, 1976.

Course Outcomes: After studying this course, the student will be able to

1. know about existence and uniqueness of solutions of IVPs.
2. apply power series methods to obtain series solutions of differential equations.
3. solve the Strum Liouville problems with eigen values and eigen functions.
4. draw the phase portraits and study the stability of solutions of linear and non-linear autonomous systems.

Course Code: MAT503	Basic Linear Algebra	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objective: The main objectives of this course are

1. To use mathematically correct language and notation for Linear Algebra.
2. To achieve computational proficiency in the procedures in Linear Algebra.

Syllabus:

Unit-I:

System of linear equations, equivalent system of linear equations, elementary row operations, row reduced Echelon form (RREF), elementary matrices, invertible matrices.

Unit-II:

Vector spaces and subspaces, algebra of subspaces, linear span, linear independence, basis and dimension of vector spaces, co-ordinate vector relative to an ordered basis.

Unit-III:

Linear transformations, rank nullity theorem, non-singular linear transformations, matrix of a linear transformation, effect of change of basis on matrix of a linear transformation.

Unit IV:

Linear functionals and dual spaces, transpose of a linear transformation, eigen values and eigen vectors, characteristic and minimal polynomial, Cayley-Hamilton theorem.

Unit-V:

Diagonalization of linear transformations and matrices, algebraic and geometric multiplicity of eigen values, invariant subspaces, triangulation of linear transformation.

Recommended Text Books:

1. Hoffman and Kunze, *Linear algebra*, 2^{nd} edition, PHI.
2. Friedberg, S.H., Insel, A.J., Spence, L.E., *Linear Algebra*, Pearson, 4^{th} edition, 2015.
3. Herstein, I.N., *Topics in Algebra*, Xerox college publishing, 2^{nd} edition.
4. Lipschutz, lipson, *Schaum's outline of linear algebra*, McGraw Hill, 3^{rd} edition.

Course Outcomes: Upon successful completion of this course, students will be able to

1. Learn the basic concepts of linear algebra including, system of linear equations, vectors spaces, linear transformations and eigen values.
2. Apply the techniques of Linear Algebra to other branch of mathematics and to model, solve, and analyze real-world situations.
3. Answer the questions related to linear algebra in NET/GATE examinations.

Course Code: MAT504	Mathematical Methods	L	P	C
		3	0	3

Pre-requisites: NIL.

Course Objectives: The main aim of this course is to introduce the various integral transforms such as Laplace transforms, Fourier transforms, and Hankel Transforms. Further, it explains Z-transforms and then Perturbation theory for solving first and second order differential equations.

Syllabus:

Unit-I: Laplace Transforms

Piecewise continuous functions, Exponential order, Sufficient conditions for L.T., Properties, Inverse Laplace transform, Convolution theorem, Application of Laplace transforms to Differential equations and Integral equations.

Unit-II: Fourier Transforms

Introduction to Fourier integrals, Fourier transforms, Properties, Fourier Sine and Cosine transforms, Convolution theorem, Finite Fourier transforms, Application of Fourier Transforms to Differential equations and Integral equations.

Unit-III: Hankel Transforms

Definition, Elementary properties, Inversion formula, Parseval Relation, Finite Hankel Transforms, Fourier-Bessel Series, Applications of Hankel Transforms to Differential Equations.

Unit-IV: Z-Transforms

Dynamic Linear Systems, Definition of Z-Transform, Summation of infinite Series, Properties, Parseval's Formula, Initial and Final value theorems, Inverse Z Transforms, Applications of Z-Transforms to Finite Difference Equations.

Unit-V: Perturbation Methods

Perturbation theory, Regular perturbation theory, Singular perturbation theory, Asymptotic matching, Time Scaling Method, Perturbation solution of first and second order differential equations.

Recommended Text Books:

1. Debnath, L. & Bhatta, D., *Integral Transforms and Their Applications*, CRC Press, 2015.
2. Neyfeh, A. H., *Perturbation Methods*, Wiley, 2004.

References:

1. Sneddon, I.N., *The Use of Integral Transforms*, Tata McGraw-Hill, 1972.
2. Baidyanath, P., *An Introduction to Integral Transforms*, CRC Press, 2018.
3. Hinch, E. J., *Perturbation Methods*, Cambridge University Press, 1991.

Course Outcomes: After studying this course, the student will be able to

1. Apply Laplace and Fourier transforms to solve IVPs and BVPs occurs in Differential equations.
2. Understand the Hankel Transforms and apply them to solve the Differential Equations.
3. Apply Z-Transforms to solve the Finite Difference Equations.
4. Apply Perturbation theory to solve first and second order differential equations.

Course Code: MAT505	Basic Mathematical Softwares Lab	L	P	C
		0	3	3

Pre-requisites: NIL.

Course Objectives: The aim of this course is to introduce various elements of Latex and beamer presentation, and basics of Scilab and Python to solve some mathematical problems.

Syllabus:

Module-I: Basics of LaTeX

Installation of LaTeX software in Windows/Linux, Creating account in Overleaf, Elements of LaTeX, writing mathematical equations into LaTeX file, Use of Packages: amsmath, amssymb, amsthm, amsfonts, graphic, etc.; Defining commands and environments, Figures and Tables, Pictures and colours, Page configurations: Title, Abstract, Keywords, Chapter, Sections and Subsections; Labelling of equations, Table of contents, List of figures, References and their citations, other document classes: Article, Report, Book, Beamer Presentation.

Module-II: Basics of Scilab

Installation of Scilab software in Windows/Linux, Introduction to Scilab, Scalars and Vectors, Matrix operations, polynomials, Plotting, Functions, Loops. String Handling Functions, Basic programming in Scilab.

Module-III: Basics of Python

Introduction to Python: Installation, Python Editors; Variables, Objects, Operators, Conditional Statements Loops: for, while, do while; Functions, building own functions, Numpy: Multi-dimensional Arrays; Matplotlib: 2D and 3D plotting in python Regular Expressions; Scipy: Scientific library for python Pandas; SymPy: Symbolic mathematics and computer algebra.

Recommended Text Books:

1. L. Lamport, *LaTeX: A Document Preparation System*, User's Guide and Reference Manual. 2nd Edition, Addison Wesley, New York, 1994.
2. Sandeep Nagar, *Introduction to Scilab: For Engineers and Scientists*, Apress Berkeley, CA, 2017.
3. Amit Saha, *Doing Math with Python*, No Starch Press, 2015.

References:

1. D. F. Griffiths and D. J. Higham, *Learning LaTeX*, 2nd Edition, Philadelphia, Pennsylvania, SIAM, 1997.
2. Tejas Sheth, Scilab: *A Practical Introduction to Programming and Problem Solving*, Createspace Independent Pub., 2016.
3. R. Thareja, *Python Programming: Using Problem Solving Approach*, Oxford HED, 2017.
4. K.V. Namboothiri, *Python for Mathematics Students*, Version 2.1, March 2013. (https://drive.google.com/le/d/0B27RbnD0q6rgZk43akQ0MmRXNG8/view)

Note: Module-I is compulsory. From Module-II and Module-III, one will be taught depending on the laboratory conditions and availability of the faculties in the department.

Course Outcomes: At the end of this course, the student will

1. develop programming skills.
2. learn basic techniques of Latex, Scilab and Python.
3. gain some expertise in developing programs to solve various mathematical problems.

M.Sc. (Mathematics): I Year II Semester

Course Code: MAT551	Basic Analysis	L	P	C
		4	0	4

Pre-requisites: Real number system, Archimedean property, density of rational and irrational numbers, finite, countable and uncountable sets.

Course Objectives: The main objectives of the course MAT-501 include studying the notions of continuity, differentiability and integrability of real valued functions, and the basic properties of metric spaces.

Syllabus:

Unit-I:
Continuity, properties of continuity of a real valued function of single variable, monotone functions, types of discontinuities of monotone functions.

Unit-II:
Differentiability, properties of differentiability of a real valued function of a single variable, mean value theorems, maxima-minima, Riemann integration, fundamental theorem of calculus.

Unit-III:
Sequence and series of functions, pointwise and uniform convergence, functions of bounded variation, Weierstrass approximation theorem.

Unit-IV:
Metric and metric spaces, open and closed sets, interior, closure, boundary of a set, convergence of a sequence, equivalent metrics, completeness, Cantor's intersection theorem.

Unit-V:
Dense sets, sets of first and second category, Baire category theorem, continuity, uniform continuity, homeomorphism, compactness, connectedness

Recommended Text Books:

1. Kumar A, Kumaresan S., *A basic course in Real Analysis*, CRC Press, Taylor and Francis.
2. Bartle, R.G., Sherbert, D.R., *Introduction to Real Analysis*, John Wiley, and Sons.
3. Simmons, G.F., *Introduction to Topology and Modern Analysis*, Tata Mc Graw Hill.
4. Runde, V. A., *Taste of Topology*, Springer.

Course Outcomes: After studying this course, the student will be able to

1. Learn the important properties of continuous real valued functions.
2. Understand the notions of differentiable and Riemann integrable functions.
3. Distinguish the notions of pointwise and uniform convergence of sequence of real valued functions.
4. Learn the basic properties of metric spaces and understand the notions of compactness and connectedness.

Course Code: MAT552	Basic Operations Research	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objectives: To understand the methodology of solving OR problems in the following areas

1. To develop formulation skills in Linear programming models and finding solutions usingRevised Simplex Method and analyze the results using Sensitivity Analysis.
2. To understand the basics in the field of game theory and to get the solutions with the real-life problems under different scenarios.
3. To know how project management techniques help in planning and scheduling a project.
4. Use of Deterministic Inventory models on applications in supply chain and manufacturing systems.
5. Integer programming models formulation for integer solutions and discuss the solution techniques for pure, mixed integer solutions using Gomory's Cutting plane method branch- and bound method.

Syllabus:

Unit-I:
Revised simplex method, post-optimal analysis.

Unit-II:
Game theory with and without saddle point, different methods of solving game theoretic problems.

Unit-III:
Job sequencing problem, Project management. PERT and CPM techniques, activities, network diagram, forward pass method, oat of activity and event, critical path.

Unit-IV:
Deterministic Inventory control models under different situations.

Unit-V:
Integer programming problems, Gomory's all integer cutting plane method, Gomory's mixed integer cutting plane method, Branch and bound technique.

Recommended Text Books:

1. Wagner, H.M., *Principles of Operations Research*, Prentice Hall.
2. Sharma. J.K., *Operations Research: Theory and Applications*, Mcmillan.
3. Man Mohan, Gupta, P.K., Swarup Kanti, *Operation Research*, S. Chand Sons.

References:

1. Shenoy, L.V., *Linear Programming: Methods and Applications.*
2. Vohra, N.D., *Quantitative Techniques in Management*, Tata McGraw Hill.

Course Outcomes: After studying this course, the student will be able to

1. Develop linear programming (LP) models.
2. Propose the best strategy using decision making methods under uncertainty and select the best strategy based on decision criteria under risk in game theory.
3. Formulate and solve problems as networks for shortest path, maximum flow, minimal spanning tree, critical path, minimum cost flow.
4. Use CPM and PERT techniques, to plan, schedule, and control project activities.
5. Set up decision models and use some solution methods.

Course Code: MAT553	Partial Differential Equations	L	P	C
		4	0	4

Pre-requisites: MAT502-Ordinary Differential Equations.

Course Objectives: The aim of this course is to introduce first and higher order partial differential equations and their classification, and analytic methods for computing the solutions of various partial differential equations. It also explains various applications of partial differential equations in physical situations like wave equation, heat equation, and Laplace equation.

Syllabus:

Unit-I: First Order P.D.E.

Surfaces and curves, Genesis of first order PDEs, Classification of integrals, Pfaffian differential equations, Lagrange's method, Compatible systems, Charpit's method, Jacobi's method, Integral surfaces passing through a given curve, Method of characteristics, Cauchy problem for quasi-linear and non-linear first order PDEs.

Unit-II: Second Order P.D.E.

Origin of second order PDE, Classification of second order PDEs, Canonical forms of Hyperbolic, Elliptic and Parabolic type PDEs, Characteristic curves, Variable separable methods for second order linear partial differential equations.

Unit-III: Wave equation

Vibrations of an infinite string, D'Alembert's solution, Vibrations of a semi-infinite string, Vibrations of a finite string, Method of separation of variables, Riemann method, Duhamel's principle for wave equation.

Unit-IV: Heat equation

Heat conduction in an infinite rod, Heat conduction in a finite rod, Existence and uniqueness of the solution, Solution by method of separation of variables, Duhamel's principle for heat equation.

Unit-V: Laplace's equation

Laplace's equation, Maximum and minimum principles, Dirichlet problem and Neumann problems for a circle, Dirichlet problem for a circular annulus, Green's function for Laplace's equation.

Recommended Text Books:

1. Amaranath, T., *An Elementary Course in Partial Differential Equations*, 2nd edition, Narosa Publishing House, 2012.

References:

1. Shankara Rao, K., *Introduction to Partial Differential Equations*, PHI Publications, 3rd Edition. 2011.
2. Tyn Myint-U, *Partial Differential Equations for Scientists and Engineers*, 3rd edition, Appleton & Lange publications, 1987.
3. Sneddon, I., *Elements of Partial Differential Equations*, McGraw Hill, NY, 1957; Dover, 2006.

Course Outcomes: On successful completion of this course, the students will be able to

1. Apply basic concepts to solve the first order PDEs and Cauchy problems of first order.
2. Determine the integral surfaces passing through a curve, characteristic curves of second order PDE.
3. Understand the formation and the solution of some significant PDEs like wave equation, heat equation and Laplace’s equation.
4. Apply the knowledge of PDEs to solve the real-life problems and understand their physical interpretation.

Course Code: MAT554	Introduction to Graph Theory	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objectives: The main objectives of this course are to
1. Introduce concepts of mathematical logic for analysing propositions and proving theorems, and basic concepts of graphs, digraphs, and trees.
2. Determine if a given graph is simple or a multigraph, directed or undirected, cyclic, oracyclic, and determine the connectivity of a graph.
3. Represent a graph using an adjacency list and an adjacency matrix and apply graph theory to application problems such as computer networks.
4. Determine if a graph has a Euler or a Hamilton path or circuit. Also, determine if a graph is a binary tree or not a tree.
5. Use the properties of trees to classify trees, and identify ancestors, descendants, parents, children, and siblings.

Syllabus:

Unit-I:

Graph, Subgraph, Verities of graphs, degree and incidence, isomorphism, intersection graph, operations on graph, walks and connectedness, trees cycles.

Unit-II:

Traversability, Eulerian and Hamiltomian graphs.

Unit-III:

Ramsey's Theorem - Chromatic Number - Brooks' Theorem - Chromatic Polynomials.

Unit-IV:

Plane and planar Graphs, Euler's Formula,Kuratowski's theorem. The Five-Colour Theorem andthe Four-Colour Conjecture.

Unit-V:

Networks: Flow, Cut, The Max-Flow Min-Cut Theorem, Menger's Theorem, Feasible flows.

Recommended Text Books:

1. J.A.Bondy and U.S.R. Murthy, *Graph Theory and Applications*, Macmillan.
2. Harary, F., *Graph Theory*, Narosa Publishing House.

References:

1. West, *Introduction to Graph Theory*, Prentice Hall of India.
2. B. Kolman, Busby, R.C. & S. Ross, *Discrete Mathematical Structures*, Prentice Hall of India.
3. Liu, C.L., *Elements of Discrete Mathematics*, Tata McGraw Hill.

Course Outcomes: At the end of this course, the students will be able to
1. Define graphs, digraphs, and trees, and identify their main properties.
2. To use tree and graph algorithms tosolve real life problems.

Course Code: MAT555	Numerical Analysis (Theory)	L	P	C
		2	0	3

Pre-requisites: MAT551-Basic Analysis and MAT505- Basic Mathematical Software Lab.

Course Objectives: The main aim of this course is to introduce the various interpolation methods, numerical differentiation, and integration. Further, it explains various numerical methods for solving algebraic equations, initial-value problems, and finite-difference methods to second order PDE.

Syllabus:

Unit-I:

Representation of integers and fractions, Fixed point and floating-point arithmetic, Error propagation, Loss of significance, Polynomial interpolation: Existence and uniqueness of an interpolating polynomial, Lagrange interpolation, Inverse interpolation, Hermite interpolation, Error of the interpolating polynomials, Piecewise-polynomial approximation (up to cubic splines).

Unit-II:

Numerical differentiation: Method of undetermined coefficients, Extremum values, Numerical integration: Newton-Cotes closed and open type formulae, Errors, Composite rules, Adaptive quadrature, Approximation of functions: Chebyshev approximation, Least square approximation and Orthogonal polynomial approximation.

Unit-III:

Solution of nonlinear equations: Bi-section method, Regula-falsi method, Fixed point iteration method, Secant method, Newton-Raphson method, Convergence of methods.

Unit-IV:

Solution of O.D.E.: Taylor series method, Explicit and implicit methods, Euler methods, Mid-point formula, Modified Euler's method and their convergence, Runge-Kutta methods (up to 2nd order O.D.E.), Predicator-Corrector methods, Finite difference methods for B.V.P. (second and fourth order).

Unit-V:

Solution of linear P.D.E. (up to second order): Derivation of difference equations for Heat equation, Wave equation, Laplace equation, Poisson equation, Consistency, Initial value problems, P.D.E. with Dirichlet and Neumann boundary conditions.

Course Code: MAT555	Numerical Analysis (Practical)	L	P	C
		0	2	3

A list of practical problems related with the Units I-V, based on some numerical methods will be studied with the use of software programmes like SCILAB/MATLAB/PYTHON.

Internal Assessment: 30 Marks
End Laboratory: 30 Marks
External Assessment (Theory): 40 Marks

Recommended Text Books:

1. Conte, S.D., and deBoor, C., *Elementary Numerical Analysis-An Algorithmic Approach*, 3rd edition, McGraw Hill, 1981.
2. Thomas, J. W., *Numerical Partial differential Equations: Finite Difference Methods*, Springer, 1998.

References:

1. Quarteroni, A., Saleri, F., and Gervasio, P., *Scientific Computing with MATLAB and Octave*, 3rd edition, Springer, 2010.
2. Sastry, S.S., *Introduction to Methods of Numerical Analysis*, 5th edition, PHI, 2012.
3. Jain, M.K., Iyengar, S.R.K., and Jain, R.K., *Numerical Methods for Scientific and Engineering Computation*, 4th edition, New Age Int., 2004.

Course Outcomes: After studying this course, the student will be able to

1. construct the interpolating polynomial to the given equally spaced and unequally spaced data.
2. compute the extremum values using numerical differentiation and understand various approximations of functions.
3. apply various iterative methods for solving the algebraic and transcendental equations.
4. solve the initial-value problems with the help of various numerical methods.
5. write the computer programme using SCILAB/MATLAB/PYTHON to test and implement various numerical methods studied in the course.

Course Code: MAT556	Evolution of Modern Indian Mathematics	L	P	C
		2	0	2

Prerequisites: NIL.

Course Objectives: The main objective of this course is to give the students brief idea about the famous Indian mathematicians and their contributions in mathematics and related branches of science.

Syllabus:

Unit-I:
Contribution of Kerala School mathematics including Madhaba.

Unit-II:
Ramanujan and his works in various branches of Mathematics.

Unit-III:
Contribution of S. N Bose, P.C. Mahalonobis, D. R. Kaprekar and S. S. Pillai.

Unit-IV:
Life and Works of C. R Rao, Sakuntala Devi, Harishchandra and Narendra Karmarkar.

Unit-V:
Works of M. Bhargava, Akshay Venkatesh, Sujata Ramdorai, Srinivasa Vardhana.

Recommended Text Books/Sources:

1. Srinivas, Sridharan, Emch, *Contributions to the history of Indian Mathematics*, Hindustan Book Agency, 2005.
2. Plofker, *Mathematics in India*, Princeton University Press, 2009.
3. Materials freely available in the internet.

Course Outcomes: On successful completion of the course, students will be able to

1. Learn about systematic development of modern Mathematics in India.
2. Learn about some great Indian mathematicians and their contribution to the world of Mathematics.

M.Sc. (Mathematics): II Year I Semester

Course Code: MAT601(A)	Topology	L	P	C
		4	0	4

Pre-requisites: MAT551-Basic Analysis.

Course Objective: The main objective of the course is to acquaint students with the concept of topological properties and other important mathematical concepts which can be generalized in topological spaces, so that they learn and appreciate the nature of abstract Mathematics

Syllabus:

Unit-I:
Topological spaces, definition and examples, open and closed sets, metrizable spaces, neighbourhoods, basis, subbases.

Unit-II:
Closure, Kuratowski closure operation, dense subsets, separable spaces, boundary and interior, continuity and homeomorphism.

Unit-III:
Compactness, product topology, Tychonoff's theorem, local compactness, one-point compactification.

Unit-IV:
Path-connectedness, connectedness, intermediate value theorem, components, totally disconnected spaces, local connectedness.

Unit-V:
Countability and separation axioms, Urysohn's lemma, Urysohn's metrization theorem, Tietze's extension theorem

Recommended Text Books:

1. Runde, V., *A Taste of Topology*, Springer.
2. Munkres, J.K., *Topology*, Prentice Hall of India.
3. Kelley, J., *Topology*, Springer.
4. Willard, S, *General Topology*, Dover Publications.

Course Outcomes: After studying this course the student will learn

1. Various types of topological spaces which are generalizations of metric spaces.
2. Understand continuity, compactness, connectedness, and homeomorphism in general setting without having the notion of a metric.
3. Techniques to identify the topological spaces which can be considered as metric spaces and how they differ from the spaces which are not generated by a metric.

Course Code: MAT601(B)	Number Theory	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objective: The main objective of this course is to introduce the basic concepts of elementary number theory with a problem-oriented approach. This will enable them to take up more advanced courses like algebraic number theory, analytic number theory and application of number theory in cryptography.

Syllabus:

Unit-I:
Divisibility, Euclidean algorithm, linear Diophantine equations, prime numbers, congruence, primitive roots, pseudoprimes.

Unit-II:
Quadratic residues, Legendre and Jacobi symbols, quadratic reciprocity law, sum of two, three and four squares.

Unit-III:
Arithmetic functions, the prime counting function, Rieman zeta function, Dirichlet's character, primes in arithmetic progression.

Unit-IV:
Diophantine equations, Pell's equation and continued fractions, Squares in the Fibonacci sequence, binary and ternary quadratic forms.

Unit-V:
Quadratic fields, algebraic number fields, algebraic integers, ideals in number fields, units, and ideal classes.

Recommended Text Books:

1. *A classical introduction to modern number theory*, Rosen, Ireland, Springer.
2. *Number Theory*, Andrej Dujella, Školska knjiga, 1st edition (April 1, 2021).

Course Outcomes: After successful completion of the course, the students will

1. Learn about prime numbers and their divisibility properties.
2. Learn about certain arithmetic functions and their connections with primes.
3. Learn about quadratic residues, various Diophantine equations and ideal classes in algebraic number fields.
4. Be equipped to take up more advanced courses in algebraic and analytic number theory leading to research in these coveted areas.

Course Code: MAT601(C)	Functional Analysis	L	P	C
		4	0	4

Pre-requisites: MAT503-Basic Linear Algebra and MAT551-Basic Analysis.

Course Objective: The main objective of this course is to study analysis in infinite dimensional vector spaces and thereby studying various properties and theorems.

Syllabus:

Unit-I:

Normed linear spaces, Banach spaces, subspaces, finite and infinite dimensional normed linear spaces, compactness, equivalent norms, Riesz's lemma.

Unit-II:

Continuity of linear maps, Hahn Banach theorem, consequences of Hahn-Banach theorem, topological dual of normed linear spaces, natural embedding, reflexive spaces.

Unit-III:

Uniform boundedness principle, Banach Steinhaus theorem, open mapping theorem, closed graph theorem and their applications.

Unit-IV:

Inner product spaces, Hilbert spaces, orthogonal complements and direct sums, orthonormal sets and sequences, series related to orthonormal sequences and sets, total orthonormal sets and sequences, Parsevals identity.

Unit-V:

Orthogonal projection, Riesz representation theorem, adjoint operators, normal operators, unitary operators and self-adjoint operators on Hilbert spaces, compact operators on Banach and Hilbert spaces.

Recommended Text Books:

1. Kreyszig, E., *Introduction to Functional Analysis with Applications*, John Wiley and Sons.
2. Limaye, B.V., *Functional Analysis*, New Age International.
3. Rynne B.R., and Youngson, M.A., *Linear Functional Analysis*, Springer.

Course Outcomes: After studying this course, the student will be able to

1. Verify the requirements of a norm, completeness with respect to a norm, relation between compactness and dimension of a space, check boundedness of a linear operator and relate to continuity.
2. Learn and appreciate four basic theorems, viz. the Hahn-Banach theorem, uniform bounded-ness principle, the open mapping theorem, and the closed graph theorem.
3. Learn the concept of Hilbert spaces and their properties which make them special in the class of all Banach spaces.

Course Code: MAT601(D)	Numerical Linear Algebra	L	P	C
		4	0	4

Pre-requisites: MAT503-Basic Linear Algebra and MAT555-Numerical Analysis.

Course Objective: This course mainly focuses on iterative techniques for solving linear systems of equations which typically stem from the discretization of partial differential equations. In addition, computation of eigenvalues, least square problems and error analysis will be discussed.

Syllabus:

Unit-I: Floating Point Numbers and Errors in Computation, Stability of algorithms and conditioning of problems, Matrix and Vector norms, Perturbation Analysis of the Linear System Problem, Properties of the Condition Number of a Matrix.

Unit-II:

LU Factorization method, Householder transformation with applications to LU, Hessenberg reduction, Orthonormal bases, and orthogonal projections.

Unit-III:

QR Factorization, Estimation of the condition numbers, ill conditioned systems, Wilkinson's algorithm for ill-conditioned systems, Classical and Modified Gram-Schmidt Algorithms for QR Factorizations.

Unit-IV:

Classical iterative methods: Jacobi, Gauss-Seidel and successive overrelaxation (SOR) methods and their convergence; Least-squares Solutions to Linear Systems: existence and uniqueness, Normal equations, Pseudo Inverse.

Unit-V:

Krylov subspace methods: GMRES, Conjugate-gradient, biconjugate-gradient (BiCG), BiCGStab methods; Eigenvalues and eigenvectors: Eigenvalue sensitivity, Arnoldi iteration, implicitly restarted Arnoldi iteration, SVD Computation.

Recommended Text Books:

1. Biswa Nath Datta, *Numerical Linear Algebra and Applications*, 2nd edition, Prentice Hall India, 2013.

References:

1. Trefethen, L. N., and David, B., *Numerical linear algebra*, Vol. 50. SIAM, 1997.
2. Watkins, D. S., *Fundamentals of Matrix Computations*, John Wiley & Sons, 2002.
3. Allaire, G., Kaber, S.M., *Numerical Linear Algebra*, Springer New York, 2008.
4. Meyer, C.D., *Matrix Analysis and Applied Linear Algebra*, SIAM, 2000.

Course Outcomes: At the end of the course, students will be able to

1. Understand the basic concepts of linear algebra related to stability, accuracy of numerical solutions.

2. Find QR factorization of a matrix using Householder transformation and study its applications.
3. Write various algorithms to solve system of linear equations to understand computational issues.
4. Describe the numerical procedure of eigenvalue problem, and apply the SVD of a matrix in solving real life problems.

Course Code: MAT602	Complex Analysis	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objectives: The course aims to familiarize the learner with complex function theory, analytic function theory, the concept of Cauchy's theorems, integral formulas, singularities, and contour integrations and to provide a glimpse of maximum modulus principle and Schwarz' lemma.

Syllabus:

Unit-I:

Algebra and topology of the complex plane, stereographic projections, continuity of complex functions, power series, analytic functions.

Unit-II:

Elementary functions, Mobius transformation, power series representation of analytic functions, zeros of analytic functions.

Unit-III:

Definition and simple properties of complex integral, Cauchy's theorem and integral formula, Morera's theorem, Cauchy's inequalities, Liouville's theorem.

Unit-IV:

Counting zeros of holomorphic function, open mapping theorem, maximum modulus principle, Schwarz's lemma, Singularities and their classification, Laurent series expansions, Casorati-Weierstrass theorem, meromorphic functions.

Unit-V:

Calculus of residues, Cauchy's residue theorem, evaluation of definite integrals using residue Argument principle, Rouche's theorem, Complex analytic proofs of fundamental theorem of algebra.

Recommended Text Books:

1. Conway, J.B., *Functions of one Complex Variable*, 2nd Edition, Narosa Publishing House.
2. Ponnusamy, S., *Foundations of Complex Analysis*, Narosa Publishing House.
3. Bak, J., Newman, D.J., *Complex Analysis*, 3rd edition, Springer.

Course Outcomes: After studying this course, the student will be able to

1. Understand analytic function as a mapping on the plane, branch of logarithm, understand Cauchy's theorems and integral formulas.
2. Learn complex integral theory and important results relating to it.
3. Learn the notion of singularities and their applications.
4. Appreciate the difference of complex Analysis with Real Analysis.

Course Code: MAT603(A)	Advanced Operations Research-I	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objectives: The main objectives of this course are to give some basic ideas of probability and its uses in solving different OR problems like queuing theory, inventory control, etc. This course provides some basic ideas on goal programming and nonlinear programming techniques.

Syllabus:

Unit-I:
Basic concept of probability and probability distributions, simulation technique and their applications.

Unit-II:
Queuing theory and its applications, introduction to the stochastic process with applications.

Unit-III:
Probabilistic inventory control: Different models.

Unit-IV:
Goal programming problem – formulation, solution, and application.

Unit-V:
Classical and nonlinear optimization techniques with different methods of solution.

Recommended Text Books:

1. Wagner, H.M., *Principles of Operations Research*, Prentice Hall India.
2. Sharma, J.K., *Operations Research: Theory and Applications*, Mcmillan.
3. Man Mohan, Gupta, P.K., Swarup Kanti., *Operations Research*, S. Chand Sons.

References:

1. Shenoy, L.V., *Linear Programming: Methods and Applications*, New Age Int.
2. Vohra, N.D., *Quantitative Techniques in Management*, Tata McGraw Hill.

Course Outcomes: After successful completion of this course, the students will develop the

1. Skill to analyze and develop probabilistic models in inventory management.
2. Skill to formulate and solve problems based on multi-objective criteria.
3. Skill to formulate and solve critical problems onQueuing theory.

Course Code: MAT603(B)	Methods of Applied Mathematics	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objective: The purpose of this course is to introduce Adomian decomposition methods and its modifications, Homotopy methods, Variational iterative methods to solve linear and nonlinear differential equations. This course further explains the variational formulation along with the basics of Finite element methods.

Syllabus:

Unit-I: Adomian Decomposition methods

Adomian Decomposition Method (ADM): Introduction to ADM, Convergence of ADM, ADM in several dimensions, Solving ODEs, PDEs, and Integral equations by using ADM, Modified ADM, and its applications.

Unit-II: Homotopy methods

Homotopy Perturbation Method (HPM): HPM algorithm, Convergence analysis; Homotopy Analysis Method (HAM): HAM algorithm, Convergence analysis, Role of auxiliary parameter, Control of convergence, Relation to ADM and HPM, Implementation of HAM to nonlinear differential equations.

Unit-III: Variational Iteration methods

Variational Iteration Method (VIM): VIM algorithm, Convergence of VIM, Applications to ordinary differential equations, solving system of differential equations using ADM.

Unit-IV: Finite element methods-I

Variational Formulation for a BVP with homogeneous and non-homogeneous boundary conditions, Rayleigh-Ritz minimization, Collocation, least squares method, Galerkin, Petrov-Galerkin methods for boundary value problems.

Unit-V: Finite element methods-II

Solution of 1-D boundary value problems by linear, quadratic, and cubic shape functions; Solution of 2-D boundary value problems by linear, quadratic, and cubic rectangular, and triangular shape functions; One-dimensional time dependent heat and wave equations.

Recommended Text Books:

1. G. Adomian, *Solving frontier problems in Physics: The decomposition method*, Kluwer Academic Publishers, 1994.
2. S. Liao, *Beyond perturbation: introduction to the homotopy analysis method*, CRC press, 2003.
3. Belal Batiha, *Variational Iteration Method and its applications*, LAP Lambert Academic Publishing, 2012.
4. J. N. Reddy, *An introduction to the Finite Element Method*, McGraw Hill, 2020.

Course Outcomes: After studying this course, the student will be able to

1. Solve the differential equations using ADM and modified ADM.
2. Apply HAM, HPM and VIM to solve nonlinear ordinary differential equations.
3. Understand the collocation methods, and solve time dependent heat and wave equations.

Course Code: MAT603(C)	Differential Geometry	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objectives: The main objectives of this course are as follows:

1. To build a solid mathematical understanding of the fundamental notions and properties of differential geometry.
2. To build sufficient geometric intuition of the subject.

Syllabus:

Unit-I:

Curves with Torsion: Space curves, their curvature and torsion, Fundamental theorem of space curves, tangent, principal normal, curvature, bi-normal, torsion, Serret-Frenet formulae, locus of centre of curvature, examples I, spherical curvature, locus of centre of spherical curvature, theorem of curve determined by its intrinsic equations, helices, spherical indicatrix of tangent etc., involutes, evolutes, Betrand curves.

Unit-II:

Envelopes Developable Surfaces: Surface, tangent plane, normal; one-parameter family of surfaces; envelope, characteristics, edge of regression; developable surfaces; osculating developable; polar developable, rectifying developable; two parameter family of surfaces, envelope, characteristic points.

Unit-III:

Curvilinear co-ordinates on a surface, fundamental magnitudes, curves on surfaces, first and second fundamental forms, Gaussian curvature, curvilinear coordinates: first order magnitudes; directions of a surface, the second order magnitudes, derivatives of N, curvature of normal section, Meusiner's theorem.

Unit-IV:

Curves on a surface and lines of curvature: Principal directions and curvatures, first and second curvatures, Euler's theorem, Dupins indicatrix, the surface X=f(x,y), surface of revolution, examples of asymptotic lines, curvature and torsion.

Unit-V:

Geodesics, Fundamental equations of surface theory, Geodesic property, equation of geodesics, surface of revolution, torsion of a geodesic.

Recommended Text Books:

1. D. Somasundaram., *Differential Geometry-A First Course*, Narosa Publishing House.
2. Bansi Lal., *Three-Dimensional Differential Geometry*, S. Chand.

References:

1. Guggenheimer, H., *Differential Geometry*, McGraw Hill.
2. Weather burn, C.E., *Differential Geometry of three Dimensions*, Cambridge University Press.

Course Outcomes: Upon successful completion of the course, students will have the knowledge and skills to

1. Explain the concepts and role of differential geometry in modern mathematics.
2. Apply appropriate techniques of differential geometry to analyse and solve complex problems.
3. Exemplify important concepts in specific cases.
4. Relate the theory, methods, and techniques of the course to solve mathematical problems.

Course Code: MAT603(D)	Calculus of variations and Integral equations	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objectives: The focus of this course will be

1. To introduce the direct methods in Calculus of Variations concerning minimizations problems.
2. On illustrating the main methods using important prototype examples.
3. To help students to gain an insight into the application of appropriate integral equations governing the behaviour of several standard problems.

Syllabus:

Unit-I:

Linear functional, Variations of functional, Continuity and differentiability of functional, Extremum of functional, Necessary condition for extremum, Fundamental lemma of calculus of variations, Euler's equation and applications, Invariance of Euler's equation, Isometric problems, Brachistochrone problem.

Unit-II:

Generalization of Euler's equations (n-dependent functions, higher order derivatives), Variational problems with subsidiary conditions, Method of variational techniques, Derivation of basic formula, Eigen value problems, Variational theory of eigenvalues, Extremum properties of eigenvalues and some important consequences, Variational problems leading to an integral equation.

Unit-III:

Classification of integral equations, Special types of kernels, Volterra integral equations: Converting IVP to Volterra integral equations, Converting Volterra integral equations to IVP, Method of successive substitution and successive approximation, Adomian decomposition method, Modified Adomian decomposition method, Series solution method, Weakly singular Volterra integral equations, Solution of a Volterra integral equation of the first kind, Laplace transform method for a difference kernel.

Unit-IV:

Fredholm integral equations: Converting BVPs to Fredholm integral equations, Converting Fredholm integral equations to BVPs, Fredholm integral equation of second kind, Finite difference methods, Method of successive substitution and successive approximation, Resolvent kernel method, Adomian decomposition and modified Adomian decomposition method, Direct computational method to solve Fredholm equations of second kind.

Unit-V:

Fredholm-integro-differential equations, Direct computation method, Adomian decomposition method, Converting to Fredholm-integral equations; Volterra-Integro-Differential equations, Series solution method, Decomposition method, Converting to Volterra integral equations; Introduction to non-linear integral equations of Volterra and Fredholm types.

Recommended Text Books:

1. Gelfand, I.M. and Fomin, S.V., *Calculus of variations*, Prentice-Hall, Inc., 1963.
2. Krasnov, M. L., Makarenko, G. I. and Kiselev, A. I., *Problems and Exercises in Integral Equations*, translated from the Russian by George Yankovsky, 1975.

References:

1. Krasnov, M. L., Makarenko, G. I. and Kiselev, A. I., *Problems and Exercises in the Calculus of Variations*, translated from the Russian by George Yankovsky, 1975.
2. Gupta, A.S., *Calculus of variations with Applications*, PHI, 2004.
3. Kanwal Ram P., *Linear Integral Equations*, 2^{nd} edition, Birkhauser Boston, MA, 1997.
4. Shanti Swarup, *Integral Equations*, Krishna Publishers, 2019.
5. Mikhlin, S. G., *Linear Integral Equations*, Dover Publications, 2020.

Course Outcomes: Upon successful completion of the course, students will

1. Understand what are the applications of functional in different problems of applied mathematics.
2. Apply the formula that determines stationary paths of a functional to deduce the differential equations for stationary paths in simple cases.
3. Use the Euler-Lagrange equation or its first integral to find differential equations for stationary paths.
4. Solve the linear and nonlinear integral equations by different methods with some problems which give rise to Integral Equations.

Course Code: MAT604	Mathematical Modelling (with Field Work)	L	F	C
		3	2	4

Pre-requisites: NIL.

Course Objectives: The main objective of this course is to build the foundational material to use mathematics as a tool to model, understand, and interpret the world around us. This will be done

1. Through studying functions, their properties, and applications to data analysis.
2. By introducing the students to some classic mathematical models and give them the opportunity to analyse, explore and extend these models to make predictions and gain insights into the underlying phenomena.
3. By providing a strong foundation for applying mathematics and modelling to many diverse applications and for research or further study.

Syllabus:

Unit-I:
Concept of mathematical modelling: Discrete and continuous models; Dimensional analysis and scaling, Effectiveness, and validity.

Unit-II:
Discrete models: Queuing systems and its applications in operations research such as study of lines and call centres and applications to communication systems, simulation of queuing systems; Optimization: Applications of linear and nonlinear models using integer programming, Network models.

Unit-III:
Models on nonlinear dynamics (ODEs), Stability with applications to forecasting and climate change.

Unit-IV:
Diffusion models- advection, convection with applications to mixing and transport phenomena.

Unit-V:
Stochastic models-random walks, Brownian motion, stochastic differential equations with applications to finance.

Internal Assessment: 30 Marks
Theory (Written): 40 Marks
Field work: 30 Marks
Fieldwork will be based on any topic of this course. Students are required to submit a detailed report of the mathematical model based on fieldwork.

Recommended Text Books:

1. Kapur, J.N., *Mathematical modelling*, New Age International.

References:

1. Burghes, D.N., *Mathematical modelling in social, management and life sciences*, Ellios Horwood and John Wiley.

2. Sergio M. Focardi, Frank J. Fabozzi, Turan G. Bali, *Mathematical Methods for Finance: Tools for Asset and Risk Management*, Wiley.
3. Giordano, F.R., and Weir, M.D., *A first course in Mathematical Modelling*, Brooks Cole.
4. Kapur, J.N., *Insight into mathematical modeling*, Indian National Science academy.
5. Bellomo and Preziosi, *Modelling Mathematical methods and Scientific computation*, CRC.

Course Outcomes: After completion of this course the students will be able to

1. Understanding of key concepts in mathematical modelling and the associated mathematical theory with an in-depth understanding of modelling and analysis.
2. Critically analyze and interpret the suitable mathematical frameworks to model real-world problems.
3. Evaluate information about area of modelling applications and judge its reliability and significance.
4. Contribute to team and group work to develop mathematical models and to perform analyses and interpretation and for the process of learning.
5. Demonstrate a sense of responsibility, ethical behavior, and independence as a learner and as an applied mathematician.

M.Sc. (Mathematics): II Year II Semester

Course Code: MAT651	Advanced Analysis	L	P	C
		4	0	4

Pre-requisites: MAT503-Basic Linear Algebra and MAT551-Basic Analysis.

Course Objective: The objective of the course is two-fold. Firstly, to introduce the vector-valued functions of several variables and to study their analytical properties. Secondly, to introduce Lebesgue's theory of integration which is more efficient than the theory of Riemann integration by introducing the concept of Lebesgue measure.

Syllabus:

Unit-I:

Derivative of a vector valued function in R^n, partial and directional derivatives, higher order partial derivatives, mean value theorem, equality of mixed partial derivatives.

Unit-II:

Taylor's theorem for functions of several variables, extremum problem with/without constraints, implicit and inverse function theorems.

Unit-III:

Extended real number system, Lebesgue outer measure, Lebesgue measurable sets, Lebesgue measure and properties, Borel and Lebesgue sigma-algebras, measurable functions, Lusin's theorem.

Unit-IV:

Simple functions, Lebesgue integration and properties, Lebesgue's monotone convergence theorem, Fatou's lemma, Lebesgue's dominated convergence theorem, comparison with Riemann integration.

Unit-V:

Uniform integrability and tightness, Vitali convergence theorem, convergence in Lebesgue measure, absolute continuity.

Recommended Text Books:

1. Moskowitz, M., *Differential Calculus in Several Variables*, World Scientific.
2. Rudin, W., *Principles of Mathematical Analysis*, McGraw-Hill International.
3. Royden, H.L. and Fitzpatrick, P.M., *Real Analysis*, Prentice Hall of India.

Course Outcomes: After studying this course, the student will be able to

1. Understand the notion of real derivative of a vector-valued function of several variables and appreciate its difference with the complex derivative of a function.
2. Learn the theory of Lebesgue measure and its applications to the theory of integration.
3. Learn the notion of convergence of a sequence of functions with respect to Lebesgue measure and compare it with other known forms of convergence.

Course Code: MAT652	Mechanics & Applied Fluid Dynamics	L	P	C
		4	0	4

Pre-requisites: Vector calculus, Tensor Analysis, Multivariable functions.

Course Objectives:

1. Students will be familiarised with post-Newtonian Mechanics, D' Alembert's Principle and its application to Lagrangian Mechanics.
2. To study mechanical systems consisting of system of particles and their dynamical equations of motions under Lagrangian Mechanics.
3. To study some basic concepts of classical fluid dynamics, and formulate the constitutive equations for viscous fluids in terms of Navier-Stokes equations.
4. To find some exact closed form (analytical) solutions of Navier-stokes equations under different physical set up.
5. To understand Prandtl's boundary layer theory and 2-D boundary layer equations and analytical methods like Blasius and Von-Karman's to integrate the Boundary layer equations.

Syllabus:

Unit-I:

Constraints and constrained motion, Generalised coordinates, D'Alembert's principle, Lagrange's equations, Hamilton's Action principle, Lagrange's equations from Hamilton's principle, Hamiltonian function, Physical significance of Hamiltonian, Hamilton's canonical system of equations, Hamilton's equations from Action principle.

Unit-II:

Canonical transformations, Generating Functions, Lagrange and Poisson's brackets as canonical invariants, Equations of motion in Poisson bracket form, 2-D motion of rigid bodies, Motion of a rigid body about an axis, Fictitious forces, Euler's dynamical equations of motion for a rigid body, Force-free motion of a rigid body, Eulerian angles.

Unit-III:

Fluid and its continuum hypothesis, Lagrangian and Eulerian methods of description, Streamline and path line, Equation of continuity in fluid motion, Euler's equations of motion for perfect fluids, Integrals of Euler's equations of motion, Bernoulli's pressure equations, Flow and circulation, Kelvins circulation theorem.

Unit-IV:

Motion in two dimensions, Lagrange's stream function, Complex potential, Sources, Sinks and doublets, Images, Viscosity and viscous fluid, Newton's law of viscosity, Navier - Stokes equations, Some exact solutions of Navier- Stokes equations: steady motion of a viscous fluid between two parallel plates, steady flow through circular cylindrical pipe and annulus, some unsteady flow cases.

Unit-V:

Dynamical similarity of flows, Reynolds number, Prandtl's boundary layer theory and boundary layer equations, Blasius solution, Von-Karman's integral equations, Momentum boundary layer thickness, Displacement thickness, Energy thickness.

Recommended Text Books:

1. Herbert Goldstein, *Classical Mechanics*, 2nd edition, Narosa Publishing House, 2018. (For Units: I, II)
2. Chorlton, F., *Text book of Fluid Dynamics*, 1st edition, CBS Publishers, New Delhi, 1985. (For Units: III, IV, V)

References:

1. Rana, N.C. & Jog, P.S., *Classical Mechanics*, 25th edition, McGraw Hill Education, 2013.
2. Lamb, H., *Hydrodynamics*, 6th edition, Cambridge University Press, 1975.
3. Taylor, J. R., *Classical Mechanics*, University Science Books, 2005.
4. Acheson, D.J., *Elementary Fluid Dynamics*, Oxford university press, 2005.

Course Outcomes: On successful completion of this course, the students will be able to

1. Apply the fundamental concepts of classical mechanics in the field of machines and flow systems including Fluid Dynamics.
2. Study related to various aspects of Classical Fluid Dynamics.
3. Understand some Laminar, Incompressible, viscous (Constant) fluid dynamic models.
4. Learn some exact analytical methods.
5. Apply in some real field applications in the area of Science and Technology.

Course Code: MAT653(A)	Commutative Algebra	L	P	C
		4	0	4

Pre-requisites: NIL.

Course Objective: The primary objective of this course will be to consolidate the students' knowledge acquired in basic abstract algebra course and introduce them to the beautiful world of commutative algebra.

Syllabus:

Unit-I:

Rings and ring homomorphism, prime and maximal ideals, nil radical, Jacobson radical, operation on ideals, prime avoidance lemma, extension and contraction of ideals, units, zero divisors and nilpotent elements in polynomial ring and power series ring.

Unit-II:

Modules and module homomorphism, submodules and quotient modules, operation on submodules, direct sum and product, finitely generated modules, Nakayama's lemma, exact sequences, tensor product of modules and simple properties.

Unit-III:

Rings and modules of fraction, localisation at a prime ideal, extended and contracted ideals in Ring of fractions, Primary ideals, primary decomposition theorems and uniqueness of primary decomposition.

Unit-IV:

Integral dependence, the going-up theorem, integrally closed integral domains, the going-down theorem, discrete valuation rings and Dedekind domain.

Unit-V:

Chain conditions, Noetherian rings, Artinian rings, Noether normalisation, Hilbert Nullstellansatz.

Recommended Text Books:

1. M.F. Atiyah, I.G. MacDonald, I.G, *Introduction to Commutative Algebra*, CRC Press, Chapters 1-9.
2. R.Y. Sharp, *Steps in Commutative Algebra*, Cambridge University Press.
3. N.S. Gopal Krishnan, *Commutative Algebra*, Orient Blackswan Pvt. Ltd.
4. Miles Reid, *Undergraduate Commutative Algebra*, LMS Students text 29.

Course Outcomes: After successful completion of this course, the students will

1. Learn basic concepts in commutative ring theory like ideals, homomorphisms and other related concepts.
2. Learn basic concepts in modules and standard results like Nakayama's lemma and tensor product of modules.
3. Learn about various important concepts like primary decomposition, localisation, integral dependence, chain conditions, etc.
4. Be equipped to take research in commutative algebra, algebraic geometry, algebraic number theory and other related areas.

Course Code: MAT653(B)	Operator Theory	L	P	C
		4	0	4

Pre-requisites: MAT601(C)-Functional Analysis.

Course Objective: To familiarize with the concept of convergence in strong and weak operator topology, spectrum of compact operators, numerical range, and related results. It also focuses on some introductory part of Banach algebra leading to the concept of C* algebra.

Syllabus:

Unit-I:

Hilbert space and its properties, adjoint operators, normal operators, unitary operators, self-adjoint operators, projection operators on Hilbert spaces.

Unit-II:

Finite rank operators on normed linear spaces, compact operators, completely continuous operators, convergence in strong and weak operator topology, cyclic vectors, invariant subspaces.

Unit-III:

Spectrum and resolvent, spectral mapping theorem for polynomials, spectral radius, subdivision of spectrum, spectrum of compact operators, numerical range, Toeplitz-Hausdorff theorem, Spectral theorems for compact operators on Hilbert spaces.

Unit-IV:

Definition and examples of Banach algebra, regular and singular elements, topological divisors of zero, radical and semi-simplicity, Gelfand mapping.

Unit-V:

Involutions in Banach algebras, the Gelfand-Neumark theorem, Banach-Stone theorem, the stone-chech compactification, introduction to commutative C*-algebras.

Recommended Text Books:

1. Conway, J.B., *A Course in Functional Analysis*, Springer.
2. Simmons, G.F., *Introduction to Topology and Modern Analysis*, Tata McGraw Hill.
3. Halmos, P.R., *A Hilbert Space Problem Book*, Springer.
4. Rudin, W., *Functional Analysis*, Tata McGraw Hill.

Course Outcomes: After studying this course, the student will be able to

1. Compare convergence of a sequence of operators with respect to norm topology, strong operator topology and weak operator topology.
2. Learn the concepts of cyclic vectors, invariant subspaces, spectrum and resolvent giving rise to spectral mapping theorem for polynomials, compute numerical radius of an operator.
3. Understand fundamentals of Banach algebra and some of the important theorems like Gelfand-Neumark theorem, Banach-Stone theorem and Stone-Check compactification.

Course Code: MAT653(C)	Cryptography	L	P	C
		4	0	4

Pre-requisites: MAT601(B)-Number theory.

Course Objective: After taking up a course in number theory in an earlier semester, the primary objective of this paper will be to introduce basic ideas in cryptography and to teach the students how to apply number theory in real life problems involving network security.

Syllabus:

Unit-I:
Time estimates for doing arithmetic, Divisibility and Euclidean algorithm, Congruence, Chinese Remainder Theorem, prime numbers.

Unit-II:
Finite field arithmetic, Quadratic residues, Legendre symbol, quadratic reciprocity law, Jacobi symbol and simple properties.

Unit-III:
Simple cryptosystems, Encyphering matrices, The idea of Public Key cryptography, RSA crypto-system, Discrete logarithm-based crypto-systems, Knapsack problem.

Unit-IV:
Primality Testing and factoring, Pseudo primes, Rho Method, Fermat factorisation and factor bases, The continued fraction method.

Unit-V:
Introduction to elliptic curves, Group structure, Rational points on elliptic curves, Elliptic Curve Cryptography. Elliptic curve primality test and factorisation, Known attacks.

Recommended Text Books:

1. N. Koblitz, *A Course in Number Theory and Cryptography*, Springer 2006.
2. Niven, H.S. Zuckerman, H.L. Montgomery, *An Introduction to theory of numbers*, Wiley, 2006.
3. L. C. Washington, *Elliptic curves: number theory and cryptography*, Chapman & Hall/CRC, 2003.

Course Outcomes: After taking up this course, the students will

1. Consolidate their understanding of certain basic concepts in elementary number theory.
2. Learn about simple cryptosystems like RSA and Discrete Logarithm problem-based cryptosystems.
3. Learn primality testing and factoring large primes, Rho's method, and continued fraction method.
4. Learn basic concepts in elliptic curves and its use in cryptography.

Course Code: MAT653(D)	Advanced Computational Methods	L	P	C
		4	0	4

Pre-requisites: MAT555-Numerical Analysis.

Course Objective: The aim of this course is to introduce various Finite Difference and Finite Volume methods for solving initial and boundary value problems. This course further explains the Spectral Methods in computing the solutions of various ordinary differential equations.

Syllabus:

Unit-I: Finite difference methods

Mixed derivatives, Finite difference formulation, Explicit and Implicit schemes, Consistency, Stability and convergence, Stability analysis by matrix method and von Neumann method, Finite difference schemes for initial and boundary value problems - FTCS, backward Euler and Crank-Nicolson schemes, ADI methods, upwind scheme.

Unit-II: Finite volume methods-I

Convection-Diffusion problems and discretisation: Steady 1D, 2D and 3D convection and diffusion problems; Discretisation schemes: Central differencing scheme, Upwind differencing scheme, Hybrid differencing scheme, Power-law scheme; Properties of discretisation schemes: Conservativeness, Boundedness.

Unit-III: Finite volume methods-II

Higher-order differencing schemes: QUICK scheme, Stability problems of QUICK scheme and remedies, Generalisation of upwind-biased discretisation schemes, Total variation and TVD schemes, Criteria, Implementation and Evaluation of TVD schemes; Solution algorithms: SIMPLE algorithm, SIMPLER algorithm, Crank-Nicolson scheme, Transient SIMPLE.

Unit-IV: Spectral Methods-I

Historical background, Introduction to spectral methods via orthogonal functions, Spectral differentiation versus Finite differences, Basic layout of spectral methods; Fourier Spectral Differentiation: Fourier approximation, Fourier spectral differentiation via differentiation matrices, Smoothness, and accuracy.

Unit-V: Spectral Methods-II

Chebyshev Spectral Differentiation: Polynomial approximation, Jacobi polynomials, Chebyshev spectral differentiation via Differentiation matrices, Smoothness, and accuracy; Boundary Value Problems: Spectral method treatment for problems involving with Dirichlet/Neumann/Robin type boundary conditions.

Recommended Text Books:

1. M. K. Jain, *Numerical Solution of Differential Equations*, 4th edition, Wiley Eastern, 2018.
2. H. Versteeg and W. Malalasekera, *An introduction to CFD: The Finite Volume Method*, 2nd edition, Pearson, 2007.
3. L. N. Trefethen, *Spectral Methods in MATLAB*, SIAM, 2000.

References:

1. M. K. Jain, S. R. K. Iyengar and R. K. Jain, *Computational Methods for Partial Differential Equations*, Wiley Eastern, 2016.
2. S.V. Patankar, *Numerical Heat Transfer and Fluid Flow*, CRC Press, 2009.
3. C. Canuto, M.Y. Hussaini, A. Quarteroni and T. A. Zang, *Spectral Methods: Fundamentals in Single Domain*, 1st edition, Springer Verlag, 2006.

Course Outcomes: After studying this course, the student will be able to

1. Apply Finite difference schemes for IVP’s and BVP’s.
2. Discretize steady and unsteady convection-diffusion problems.
3. Solve convective problems using upwind, QUICK and hybrid schemes.
4. Understand the basics of spectral methods.
5. Solve IVP's and BVP's using spectral methods.

Course Code: MAT653(E)	Advanced Operations Research-II	L	P	C
		4	0	4

Pre-requisites: MAT552- Basic Operations Research and MAT602(A) - Advanced Operations Research-I.

Course Objective: The main objectives of the Advanced Operations Research- II are to analyze different complex situations in the industrial/ business scenario involving decision making with multi-objective criteria. This course discusses different advanced linear and nonlinear programming techniques such as goal programming, separable programming, dynamic programming etc. Some important results and properties of decision theory is also included in this course to develop decision making skill among the students.

Syllabus:

Unit-I:
Theory of replacement problems and their applications.

Unit-II:
Decision theory under different situations.

Unit-III:
Goal Programming: Some advanced topics (Variants of goal programming with different methodologies).

Unit-IV:
Separable programming, Geometric Programming.

Unit-V:
Introduction, mathematical formulation and some basic results on Fractional Programming, Dynamic Programming.

Recommended Text Books:

1. Wagner, H.M., *Principles of Operations Research*, Prentice Hall.
2. Sharma, J.K., *Operations Research: Theory and Application*, Mcmillan.
3. Man Mohan, Gupta, P.K., Swarup Kanti., *Operations Research*, S. Chand Sons.
4. Sharma, S. D., *Operations Research Theory, Methods and Applications*, Kedar Nath Ram Nath publisher.

References:

1. Shenoy, L.V., Linear Programming: Methods and Applications, New Age Int.
2. Vohra, N.D., Quantitative Techniques in Management, Tata McGraw Hill.

Course Outcomes: After successful completion of this course, the students will develop the

1. Skill to analyze and solve many industrial and scientific problems based on multi-objective criteria.
2. Skill to make critical industrial decision by applying the knowledge gathered from this course.

Course Code: MAT653(F)	Bio-Mathematics	L	P	C
		4	0	4

Pre-requisites: System of Differential equations, Mathematical Modelling.

Course Objective: The main objectives of this course Bio-Mathematics are

1. To introduce students to the basic mathematical tools and techniques used in biological research, such as calculus and modelling.
2. To provide students with a foundation in the principles of biology, including epidemiology, and physiology.
3. To develop students' ability to use mathematical models to describe and analyze biological systems and processes, such as population growth, and gene regulation.
4. To teach students how to design and carry out experiments to test mathematical models of biological systems.
5. To develop students' critical thinking and problem-solving skills through the application of mathematical and biological concepts to real-world problems.

Syllabus:

Unit-I:

Autonomous Differential equations, Linear autonomous systems, existence, uniqueness and continuity of solutions, diagonalization of linear systems, fundamental theorem of linear systems, Characterization of equilibrium points and various stabilities, the phase space and phase-plane of linear autonomous systems.

Unit-II:

Linearization of non-linear autonomous systems (two, three and higher dimension), non-linear equilibrium points, Stability: (a) asymptotic stability (Hartman's theorem), (b) global stability (Liapunov's second method), Periodic Solutions (Plane autonomous systems): Translation property, limit set, attractors, periodic orbits, limit cycles.

Unit-III:

Qualitative analysis of continuous models: Steady-state solutions, stability, and linearization, Routh- Hurwitz Criteria, phase plane methods, and qualitative solutions, Lyapunov second method for stability, 1-D bifurcations (saddle-node, transcritical and Hopf).

Unit-IV:

Simple situations requiring mathematical modelling, techniques of mathematical modelling, Classifications, Characteristics and limitations of mathematical models, and some simple illustrations. Mathematical modelling in dynamics through ordinary differential equations of the first order, linear growth and decay models, and non-linear growth and decay models.

Unit-V:

Predator-Prey models: Lotka-Volterra model, Gause model, Kolmogorov model, Two species competition. Epidemic Models: Deterministic model of the simple epidemic, Infection through vertical and horizontal transmission, general epidemic- Karmac-Mackendric Threshold Theorem, SI, SIR, SIRS models.

Recommended Text Books:

1. D.W. Jordan, P. Smith, *Nonlinear Ordinary Differential Equations: An Introduction to Dynamical Systems*, Oxford University Press.
2. J. Pastor, *Mathematical Ecology of Populations and Ecosystems*, Wiley-Blackwell.

References:

1. J.D. Murray, *Mathematical Biology (Vol-I, Vol-II)*, Springer- Verlag.
2. M. Kot, *Elements of Mathematical Ecology*, Cambridge University Press.

Course Outcomes: At the end of this course,

1. Students will be able to understand the mathematical concepts and techniques used in biological sciences and apply them to solve problems in the field of biology.
2. Students will be able to use mathematical modelling to describe biological phenomena, such as population growth, gene expression, and metabolic pathways.
3. Students will be able to analyze and interpret data from biological experiments and studies using mathematical tools and techniques.