CHAPTER – 5

RANDOM VARIABLE AND MATHEMATICAL EXPECTATION

Random Experiment

All the experiments that are performed can be classified into two broad divisions viz. random experiment and deterministic experiments. Deterministic experiments are those experiments in which the outcome of the experiment remains same whenever it is performed. But in case of a random experiment the outcome is found to vary each time the experiment is performed. Here the experimenter may know the set of all possible outcomes of the random experiment but cannot say with certainty which outcome will occur when the experiment is performed.

For example: Throwing of a die is a random experiment, as its outcome cannot be expressed with certainty. But, one knows the set of all possible outcomes i.e. 1,2,3,4,5 and 6.

Random Variable

The various outcomes of a random experiment is denoted with the help of a variable which is called a random variable.For example: In case of throwing a die, we may use a variable X for representing the out come of the throw. Thus X will take the values 1, 2, 3, 4, 5 and 6.

But in some cases the outcomes may be qualitative *e.g.* tossing of a coin which may be head or tail, the colours of balls drawn from an urn may be red, yellow, white etc. But for mathematical convenience the qualitative outcomes may be expressed in quantitative forms. For example, in tossing of a coin we may denote the outcome 'Head' by 1 and 'Tail' by 0. In this way each outcome of a random experiment, whether it is qualitative or quantitative, can be expressed by a real number.

The real number, which is associated with the outcome of random experiment, is called a random variable**.** The random variable takes certain values depending on chance, so it is also called as a chance variable or a stochastic variable.

An alternate definition of random variable:

Let S be a sample space corresponding to a random experiment. That is S consists of a set of all possible outcomes of the random experiment. Let s be any sample point. So $s \in S$. If for every outcome of *s* of a sample space S there is a real number denoted by X (s), X is called a function defined on S.

Thus, *A real valued function X, defined on a sample space S, of a random experiment, is called a random variable which assigns to each sample point, one and only one real number X (s) = x (say) where* $s \in S$.

Types of Random Variable

There are two types of random variables:

(a) Discrete random variable (b) Continuous random variable.

(a) Discrete random variable: If a random variable X assumes only a finite number or countably infinite number of values, then it is called a discrete random variable. The random variable X is said to take finite values only if the possible values of X are $x_1, x_2, \dots, x_n$ and is said to be countably infinite if X takes the values $x_1, x_2, \dots$.

Example: Number of throws of a fair coin before the first head occurs.

(b) Continuous random variable: If a random variable is such that it assumes any value with in a given interval, then it is called as a continuous random variable. In other words if a random variable can take infinite number of values within a given interval , $a \le x \le b$ (say) then it is called a continuous random variable.

Example: The heights of the persons collected from a crowd.

Probability Distribution

The distribution obtained by taking the possible values of a random variable together with their respective probabilities is called a probability distribution. A probability distribution can be presented either with the help of a function or in tabular form where values of the random variable and corresponding probability are shown. The probability distribution for a discrete random variable is called as a discrete probability distribution or 'probability mass function' (*pmf*) and that of a continuous random variable is called a 'probability density function' or (*pdf*).

Discrete Probability Distribution

Let X be a discrete random variable which takes the values x_1, x_2,..., x_n with respective probabilities p_1, p_2, …, p_n . Then the probability distribution of the discrete random variable X or the probability mass function is given by,

$f(x_i) = P(X = x_i) = p_i$, where, $i = 1,2,…,n$

However, $f(x_i)$ has to satisfy the following properties.

(i) $f(x_i) \geq 0$ for all i and (ii) $\Sigma f(x_i) = 1$ where $i = 1,2,…,n$

In case the discrete random variable is countably infinite then the pmf is given by,

$f(x_i) = P(X = x_i) = p_i$, where, $i = 1,2,…$

and $f(x_i)$ satisfies the following properties (i) $f(x_i) \geq 0$ for all i

and (ii) $\Sigma f(x_i) = 1$ where $i = 1,2,…$

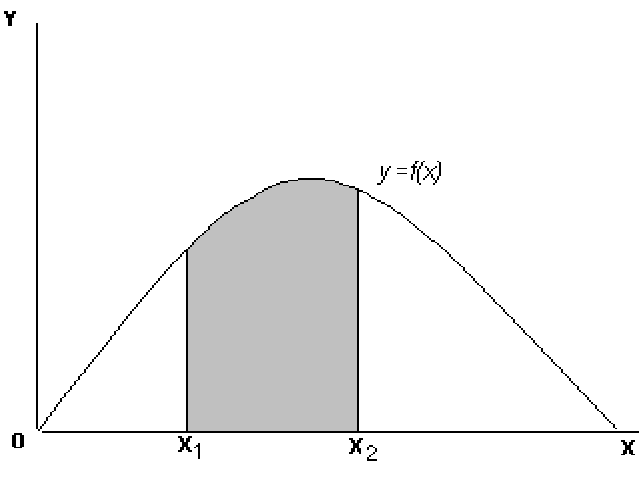
Example: Let us consider the experiment of throwing a die. If X represents the face value which turns up when a die is thrown. Then X takes the values 1, 2, 3, 4, 5 and 6 each having probability 1/6. This can be written as

P(X = x) = 1/6 where x = 1, 2, 3, 4, 5 and 6

Continuous Probability Distributions

The probability distribution of a continuous random variable X is defined by the functional notation $f(x)$, is called the probability density function or simply the density function. A probability density function is constructed in such a way that the area under its curve bounded by the X-axis is 1, when computed over the entire range of X. In case of a continuous random variable it is not possible to find the probability of the distribution at a particular point but one can find the value of the function between two points $X = x_1$ and $X = x_2$ (say). In the figure below the probability that X lies between x_1 and x_2 is given by the shaded area under the curve $y = f(x)$ lying between the ordinates $X = x_1$ and $X = x_2$.

i.e. Probability that X assumes a value between x_1 and x_2 = P ($x_1 \leq X \leq x_2$) = $\int_{x_1}^{x_2} f(x)dx$



The total area of the curve is equal to 1, that is if $a \leq X \leq b$ then we have $\int_a^b f(x)dx$

Example: A continuous random variable X is said to follow the following probability law

$f(x) = \dfrac{1}{\theta}$ where $0 \leq x \leq \theta$ is an example of a pdf.

Mathematical Expectation

Let us consider a discrete random variable X which assumes the values x_1, x_2,..., x_n with respective probabilities p_1, p_2, …, p_n , such that $\Sigma p_i = 1$, then the mathematical expectation of the random variable X is given by the sum of the products of the different values of X with their corresponding probabilities. The expectation of a random variable is generally denoted by E(X).

Thus, E(X) = $\sum_{i=1}^{n} x_i \times P(X = x_i) = \sum_{i=1}^{n} p_i x_i$ provided the series is convergent and $\Sigma p_i = 1$.

In case the discrete random variable takes countably infinite number of values then we have

$$E(X) = \sum_{i=1}^{\infty} x_i \times P(X = x_i) = \sum_{i=1}^{\infty} p_i x_i$$

If X is a continuous random variable with probability density function $f(x)$, $-\infty < x < \infty$ Then the mathematical expectation of the random variable X is given by

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx \text{ provided } \int_{-\infty}^{\infty} f(x) dx = 1$$

The expectation of the random variable X serves as the measure of central tendency of the probability distribution of X.

✍ **Note**

If g(x) is a function of the random variable X then the expectation of g(x) is given by

$$E[g(x)] \quad \sum_{i=1}^{\infty} P(X = x_i) \times g(x_i) = \sum_{i=1}^{\infty} p_i \times g(x_i) \qquad = \text{for discrete case}$$

$$= \int_{-\infty}^{\infty} g(x) \cdot f(x) dx \text{ for continuous case}$$

Remember that the range of the integral and summation depends on the range in which the variable is defined.

Some Theorems on Mathematical Expectations

Theorem 1. If C is a constant then E(C) = C

Proof: From the definition of mathematical expectation we know that

$E(C) = \Sigma C\, P[X = x_i]$

$= C\, \Sigma\, P[X = x_i] = C \times 1 = C$ | since, $\Sigma\, P[X = x_i] = 1$

Theorem 2. If C is a constant then E(CX) = C E(X).

Proof: $E(CX) = \Sigma C\, x_i\, P[X = x_i]$

$= C\, \Sigma x_i\, P[X = x_i] = C\, E(X)$

Theorem 3. If a and c are two constants then $E(aX + c) = a\, E(x) + c$

Proof: $E(aX + c) = E(aX) + E(c)$

$= a\, E(X) + c$ | Using theorem 1 and 2

Theorem 4. The expectation of the sum of two random variables are equal to the sum of their expectations i.e. E (X + Y) = E(X) + E(Y)

Corollary: The expectation of the difference of two random variables are equal to the difference of their expectations i.e. E (X –Y) = E(X) – E(Y)

Theorem 5: If X and Y are two independent random variables then,

E(XY) = E(X) E(Y)

Variance of a Random Variable

Variance is an important characteristic of a random variable. It is a measure of dispersion of the random variable. Variance of X, is denoted by Var(X) or by σ_x^2 . It is defined as the expected value of the square of the deviation of the random variable from its mean value.

$$Var(X) = E[X - E(X)]^2.$$

The positive square-root of the variance is called the standard deviation and is denoted by σ_x

A simplified expression for the variance can be derived as

$Var(X) = E[X - E(X)]^2$

$= E[X^2 - 2X. E(X) + E^2(X)]$

$= E(X^2) - 2\,E(X).\,E(X) + E^2(X)$

$= E(X^2) - 2\,E^2(X) + E^2(X)$

$= E(X^2) - E^2(X)$

✍ **Note**

It may be seen that Var(X) being the expectation of a squared quantity is always non–negative i.e.

$Var(X) \geq 0 \Rightarrow E(X^2) - E^2(X) \geq 0$

$\Rightarrow E(X^2) \geq E^2(X)$

Theorem 1: If C is a constant then Var(C) = 0

Proof: From the definition of variance we have,

$Var(C) = E[C - E(C)]^2$

$= E[C - C]^2$ | Since, $E(C) = C$

$= E(0) = 0$

Theorem 2: If C is a constant then we have $Var(CX) = C^2\,Var(X)$.

Proof: From the definition of variance we have,

$Var(CX) = E[CX - E(CX)]^2$

$= E[CX - CE(X)]^2$

$= C^2\,E[X - E(X)]^2$ | Since, $E(CX) = C\,E(X)$

$= C^2\,Var(X)$

Theorem 3: If A and C are two constants then $Var(AX + C) = A^2\,Var(X)$

Proof: We have

$Var(AX + C) = Var(AX) + Var(C)$

$= A^2\,Var(X) + 0$ |Using theorem 1 and 2

$= A^2\,Var(X)$

SOLVED ILLUSTRATION

Illustration 1: Three perfect dice are thrown. Find the expected value of the sum of the face values in the dice.

Solution: Let X be the random variable which denotes the face value of the first die. Thus we have

$X = x$:	1	2	3	4	5	6
$P(X = x)$:	1/6	1/6	1/6	1/6	1/6	1/6

So, $E(X) = \Sigma x\,P(X = x)$

$= 1\times 1/6 + 2\times 1/6 + 3\times 1/6 + 4\times 1/6 + 5\times 1/6 + 6\times 1/6$

$= 1/6\,(1+2+3+4+5+6) = 7/2 = 3.5$

Let Y and Z be the face value of the second and third die respectively.

So, $E(Y) = 3.5$ and $E(Z) = 3.5$

So we have,

$E(X+Y+Z) = E(X) + E(Y) + E(Z) = 3.5 + 3.5 + 3.5 = 10.5$

Illustration 2: Let X be a random variable with probability distribution

X:	0	1	2	3
$f(x)$:	1/3	1/2	0	1/6

Find the expectation of X, X^2 and $(X-1)^2$. Also find the variance of X.

Solution:

$E(X) = \Sigma x_i P(X = x_i)$

$= 0\times 1/3+1\times 1/2+2\times 0+3\times 1/6 = 1$

We know that

$E[g(x)] = \Sigma g(x_i) P(X = x_i)$ Here $g(x) = x^2$

Thus, $E(X^2) = \Sigma x_i^2 P(X = x_i) = 0^2\times 1/3+1^2\times 1/2+2^2\times 0+3^2\times 1/6$

$= 1\times 1/2+9\times 1/6 = 2$

$\therefore E(X^2) = 2$

For the next case we have $g(x) = (x-1)^2$

Thus, $E[(X-1)^2] = \Sigma (x_i-1)^2 P(X = x_i)$

$= (0-1)^2\times 1/3+(1-1)^2\times 1/2+(2-1)^2\times 0+(3-1)^2\times 1/6$

$= (-1)^2\times 1/3+(0)^2\times 1/2+(1)^2\times 0+(2)^2\times 1/6$

$= 1/3 + 2/3 = 1$

Now, $Var(X) = E(X^2) - [E(X)]^2$

$= 2 - 1 = 1$

Illustration 3. A man plays a gamble in which the fate of the game is decided by tossing a coin. The man wins one unit of money if a head occurs and looses 2 units of money if a tail occurs. He plays the game twice. Find his expected earnings. Also find the variance.

Solution: Here in two games there are four possibilities. Let X be the random variable, which represents the earning of the gambler. Let p_i represents the corresponding probability. They are tabulated below:

Possibility	Earnings(x_i)	Probability(p_i)
Wins both games	1 + 1 = 2	½ × ½ = 1/4
Looses first, wins second	–2 + 1 = –1	½ × ½ = 1/4
Wins first, looses second	1 +(– 2) = –1	½ × ½ = 1/4
Looses both games	–2+(–2) = –4	½ × ½ = 1/4

Thus, the expected earning of the man is given by

$E(X) = \Sigma x_i P(X = x_i)$

$= 2\times 1/4+(-1)\times 1/4+(-1)\times 1/4+(-4)\times 1/4$

$= 1/4\,(2-1-1-4) = -1$

That is the man is expected to loss one unit of money in this game.

$E(X^2) = \Sigma x_i^2 P(X = x_i) = 2^2\times 1/4+(-1)^2\times 1/4+(-1)^2\times 1/4+(-4)^2\times 1/4$

$= 1+ 1/4+1/4+ 4 = 5.5$

So, $Var(X) = E(X^2) - [E(X)]^2$

$= 5.5 - 1 = 4.5$

Illustration 4: A discrete random variable X is defined as follows:

X:	0	1	2	3	4
$P(X=x)$:	K	$3K$	0.2	K	$2K+0.1$

Find the following:

(i) the value of K (ii) find the probability distribution (iii) P(X>2) (iv) E(X) (v) V(2X – 4)

Solution: (i) We know that for a discrete random variable the probability mass function is such that

$\sum P(X=x) = 1$

Thus we have,

$P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4) = 1$

$\Rightarrow K + 3K + 0.2 + (K) + (2K+0.1) = 1$
$\Rightarrow 7K + 0.3 = 1$
$\Rightarrow K = 0.7/7 = 0.1$

(ii) So, the probability distribution becomes:

X :	0	1	2	3	4
$P(X=x)$:	0.1	0.3	0.2	0.1	0.3

(iii) $P(X>2) = P(X=3) + P(X=4) = 0.1 + 0.3 = 0.4$

(iv) $E(X) = \Sigma x\, P(X = x)$
$= 0\times 0.1 + 1\times 0.3 + 2\times 0.2 + 3\times 0.1 + 4\times 0.3 = 0.3 + 0.4 + 0.3 + 1.2 = 2.2$

(v) Now
$V(2X-4) = 2^2\, V(X) = 4\{E(X^2) - [E(X)]^2\}$ (a)

Thus,
$E(X^2) = \Sigma x^2\, P(X = x)$
$= 0^2\times 0.1 + 1^2\times 0.3 + 2^2\times 0.2 + 3^2\times 0.1 + 4^2\times 0.3 = 0.3 + 0.8 + 0.9 + 4.8 = 6.8.$

Using this result and the value of E(X) in (a) we have,
$V(2X-4) = 4\{E(X^2) - [E(X)]^2\}) = 4\{6.8 - [2.2]^2\} = 4 \times 1.96 = 7.84$

Illustration 5: A random variable X takes only two values either 0 or 1. It takes the value 1 with probability p and the value 0 with probability $q = 1 - p$. Find the expectation and variance of the random variable.

*Solution: B*y the question we have,

X:	0	1
$P(X=x)$:	q	p

Now, $E(X) = \Sigma x\, P(X = x) = 0\times q + 1\times p = p$
Also, $E(X^2) = \Sigma x^2\, P(X = x) = 0^2\times q + 1^2\times p = p$
Thus, $V(X) = E(X^2) - [E(X)]^2 = p - p^2 = p(1-p)$
So, we have expectation of the random variable is p and its corresponding variance $p(1-p)$.

Illustration 6: In a box there are n tickets, numbered serially from 1 to n. The box is shaken properly and a ticket is drawn from it. The face value of the ticket is noted. Find the (i) Expected value (ii) Variance of the face value.

Solution: Let X be a random variable that represents the face value of the ticket drawn from the box. So, the value of X may vary from 1 to n. Now, each ticket is equally probable. Since there are n tickets so the probability of the ticket having a particular face value is 1/n.
Thus, we have $P(X=x) = 1/n, \quad x = 1,2,\ldots,n$
So,
$$E(X) = \Sigma x\, P(X = x) = \Sigma x\times(1/n) = (1/n)(1+2+\ldots+n) = \frac{1}{n}\times\frac{n(n+1)}{2} = \frac{n+1}{2}$$
Now,
$$E(X^2) = \Sigma x^2\, P(X = x) = \Sigma x^2\times(1/n) = (1/n)(1^2+2^2+\ldots+n^2)$$
$$= \frac{1}{n}\times\frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{2}$$

Thus, $V(X) = E(X^2) - [E(X)]^2 = \dfrac{(n+1)(2n+1)}{2} - \left[\dfrac{n+1}{2}\right]^2 = \dfrac{(n+1)(3n+1)}{4}$

Illustration 7: The probability density function of a continuous random variable Y is given by

$f(y) = K e^{-5y}, y \geq 0$

Find (i) The value of K *(ii) E(Y) (iii) P(Y =1) (iv) P(Y≥5) (v) V(Y)*

Solution: We have, $f(y) = K e^{-5y}, y \geq 0$

(i) We know that for a pdf *f(y)*

$$\int_0^\infty f(y)dy = 1$$

$$\Rightarrow \int_0^\infty Ke^{-5y}dy = 1 \Rightarrow K\int_0^\infty e^{-5y}dy = 1 \Rightarrow \left[K\frac{e^{-5y}}{-5}\right]_0^\infty = 1$$

$$\Rightarrow K\left[\frac{e^{-\infty}}{-5} - \frac{e^0}{-5}\right] = 1 \Rightarrow K\left[0 + \frac{1}{5}\right] = 1 \Rightarrow K = 5$$

(ii) Now, $E(Y) = \int_0^\infty yf(y)dy = \int_0^\infty y5e^{-5y}dy = 5\int_0^\infty ye^{-5y}dy = 5\times\frac{1}{5^2} = \frac{1}{5}$

[Since $\int_0^\infty ye^{-5y}dy = \frac{1}{5^2}$]

(iii) P(Y=1) = 0 .

Since, the distribution is continuous, so it is not possible to compute the probability of the random variable at a point.

(iv) P[Y ≥ 1] = $\int_1^\infty f(y)dy = \int_1^\infty 5e^{-5y}dy = 5\int_1^\infty e^{-5y}dy = \left[5\frac{e^{-5y}}{-5}\right]_1^\infty = e^{-5} - e^{-\infty}$

= 0.00673 – 0 = 0.00673

(v) In order to calculate V(Y) we first calculate $E(Y^2)$

$E(Y^2) = \int_0^\infty y^2 f(y)dy = \int_0^\infty y^2 5e^{-5y}dy = 5\int_0^\infty y^2e^{-5y}dy = 5\times\frac{2}{5^3} = \frac{2}{25}$

[Since $\int_0^\infty ye^{-5y}dy = \frac{1}{5^2}$]

Thus, $V(Y) = E(Y^2) - [E(Y)]^2 = (2/25) - (1/5)^2 = 1/25$

Illustration 8. A continuous random variable X has the following p.d.f

$f(x) = x/2, \quad 0 \leq x \leq 1$

$= ½, \quad 1 < x \leq 2$

$= (3-x)/2, \quad 2 < x \leq 3$

Find the mean of the distribution.

Solution: We know that the expectation of a continuous random variable X is given by,

Here, E(X) = $\int_{-\infty}^{\infty} xf(x)dx$

Now, E(X) = $\int_0^3 xf(x)dx = \int_0^1 xf(x)dx + \int_1^2 xf(x)dx + \int_2^3 xf(x)dx$

$$= \int_0^1 x\frac{x}{2}dx + \int_1^2 x\frac{1}{2}dx + \int_2^3 x\frac{3-x}{2}dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_0^1 + \frac{1}{2}\left[\frac{x^2}{2}\right]_1^2 + \frac{1}{2}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_2^3$$

$$= \frac{1}{2}\left[\frac{1}{3}\right]+\frac{1}{2}\left[\frac{4}{2}-\frac{1}{2}\right]+\frac{1}{2}\left[\frac{27}{2}-\frac{27}{3}-\frac{12}{2}+\frac{8}{3}\right]=\frac{1}{6}+\frac{3}{4}+\frac{7}{12}=\frac{18}{12}=1.5$$

EXERCISES

Theoretical

1. Define 'random variable'. How do you distinguish between 'discrete' and 'continuous' random variables? Illustrate your answer with examples?
2. What do you mean by 'discrete probability distribution' and 'probability mass function'? Give examples of each.
3. What is meant by "expectation of a random variable'? Explain as clearly as you can.
4. Define 'expectation' and variance of a random variable. Prove that the expectation of the sum of two discrete random variables is the sum of their expectations.
5. State the 'sum law of expectation' and the 'product law of expectation' relating to two independent random variables.
6. State and prove the properties of mathematical expectation of a random variable.
7. State and prove the properties of variance of a random variable.

Problems

8. X is a discrete random variable having the following probability mass function:

X:	0	1	2	3	4	5	6	7
P(X=x) :	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Determine the constant k. (ii) Find P(X<6) (iii) Find P(X≥6) [Ans: k = 1/10, 0.81, 0.19]

9. A random variable has the following probability distribution :

X:	4	5	6	8
P(X=x) :	0.1	0.3	0.4	0.2

Find the expectation and standard deviation of the random variable. [Ans: 5.9, 1.22]

10. For what value of a will the function

f(x) = ax; x = 1,2,3,…,n

be a probability mass function of the discrete random variable X? Find the mean and variance of X.
[Ans: 2/n(n+1), (2n+1)/3, (n −1)(n+2)/18]

11. Obtain the expectation of the number of tosses before the first head in an infinite series of tosses with the same coin. [Ans: q/p]
12. If a discrete random variable X assumes the values 8, 9, 11, 15, 18, 20. Find the probabilities P(x = 9), P(x = 12), P(x < 15), P(x ≤ 15), P(x > 15), P(|x – 14| < 5). [Ans: 1/6, 0, ½, 2/3, 1/3, ½]
13. Thirteen cards are drawn from a pack of 52 cards. If aces count 1, face cards 10 and others according to denomination, find the expectation of the total score in the 13 cards. [Ans: 85/13]
14. In four tosses of a coin, let X be the number of heads. Calculate the expected value of X.
[Ans: 2]
15. Ball are taken one by one from an urn containing a white and b black balls until the first white ball is drawn . show that the expectation of the number of black balls preceding the first white ball is b/(a+1).
16. Find the expected value of the product of points on n dice. [Ans: $(7/2)^n$]
17. A continious random variable X has the pdf *F(x) = (1/2) − ax; 0≤x ≤4.* Where *a* is a constant.
(i) Determine the value of a (ii) Find the probability that x lies between 2 and 3. [Ans: 1/8, 3/16]
18. Show that *f(x) = x; 0≤x ≤1*

= k −x, 1≤x ≤2

= 0, elsewhere.

Is a probability density function for the suitable value of the constant k. [Ans: 2]

19. A continuous random variable X has the following density function,

$f(x) = 2e^{-2x}$, $x > 0$

= 0, otherwise.

Find (a) E(X) (b) $E(X^2)$ (c) V(X) [Ans: ½, ½, ¼]

20. Three coins whose faces are numbered 1 and 2 are tossed. What is the expectation of the total value of the numbers on their faces? [Ans: 4.5]

21. Find the expectation of a discrete random variable X whose p.m.f is given by $f(x) = (1/2)^x$ for $x = 1,2,3,\ldots$ [Ans: 2]

22. Find the mean and variance with probability function

$$f(x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1} \text{ for } x = 1, 2, 3, \ldots$$
$$= 0 \quad \text{otherwise}$$
[Ans: 3/2, 27/4]

23. Find the expectation of the number of failures preceeding the first success in a series of independent trials with constant probability of success *p*. Also find the standard deviation.
[Ans: $(1-p)/p$, $\sqrt{q}/p$]

24. Thirtten cards are drawn simultaneously from a pack of 52 cards. If aces count 1, face cards 10 and others according to denominator. Find the expectation of the total score on the 13 cards.
[Ans: 85/13]

25. In four tosses of a fair con if X denodes the number of heads. Calculate the expected values of X.
[Ans: 2]

26. Find $E(X), E(X^2), E[(X-\bar{X})^2]$ for the following probability distribution

X	*8*	*12*	*16*	*20*	*24*
P(X=x)	*1/8*	*1/6*	*3/8*	*¼*	*1/12*

27. A box contains *a* white and *b* black balls until the first white ball are drawn. Show that the expectation of the number of black balls preceding the first white ball is $b/(a+1)$.

28. A box contains *a* white and *b* black balls, *c* balls are drawn. Show that the expectation of the number of white balls drawn is $ca/(a+b)$.

29. Let X denotes the profit that a man can make in business. He can urn Rs. 2800 with probability ½, he may loose Rs. 5000 with probability 3/10, and he may neither lose nor gain with probability 1/5. Show that the mathematical expectation is –100.

30. If X is a random variable for which E(X) = 10 and Var(X) = 25. Find the positive values of *a* and *b* such that $y = ax - b$ has expectation *0* and variance 1.

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